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CHAOTIC DYNAMICS OF ELASTIC-PLASTIC BEAMS

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During the last decades great attention has been turned to chaotic vibrations of elastic beams, but not much has been done in the case of elastic-plastic deformations. Evidently the first paper in this field belongs to Symonds and Yu (1985), who considered the following problem. A fixed ended beam is subjected to short intensive pulse of transverse loading that produces plastic deformation. Since the ends of the beam are fixed membrane forces must be taken into account. Solving the equations of motion Symonds and Yu found that permanent deflection may be in direction opposite to the load. This phenomenon was investigated in several papers by Symonds and his collaborators. It turned out that the permanent defflection is very sensitive to small changes of load. Fractality and self-similarity which are characteristic to chaotic processes, were demonstrated. For the similarity dimension the value 0,78 and for the correlation fractal dimension ~1,44 were obtained.

In most papers of Symonds and his collaborators the real beam was replaced by a Shanley-type model with one or two degrees of freedom. In some papers (see Symonds and Qian, 1996) the Galerkin method for $2DoF$ was used; some simplifying assumptions (sandwich beam, perfectly plastic material, disregard of axial inertia forces) were made.

The aim of this paper is to present a method of solution, which is based on the Galerkin technique and is applicable for beams with an arbitrary number of DoF. Basic elements of this method were worked out by Lepik (1994, 1995).

To be brief we shall consider beams with a rectangular cross-section, B, h and L are the width, thickness and length of the beam, respectively. The equations of motion are

$$\begin{aligned} \Phi_1 &= \frac{\partial T}{\partial x} - \rho Bh \frac{\partial^2 u}{\partial t^2} = 0, \\ \Phi_2 &= \frac{\partial^2 M}{\partial x^2} + \frac{\partial}{\partial x}\left(T \frac{\partial w}{\partial x}\right) - p(x,t) - \rho Bh \frac{\partial^2 w}{\partial t^2} = 0, \end{aligned} \tag{1}$$

where ρ is density, u - axial displacement, w - deflection. Axial force T and bending moment M we shall calculate from the formulae

$$T = \int_{-h/2}^{h/2} \sigma(x,z)\, dz, \quad M = \int_{-h/2}^{h/2} \sigma(x,z) z\, dz. \tag{2}$$

We shall assume that the beam material has linear strain-hardening; elastic unloading and secondary plastic loading effects are also taken into account.

To the equations (1) we shall apply the Galerkin procedure

$$\int_0^L \Phi_1 \delta u\, dx = 0, \quad \int_0^L \Phi_2 \delta w\, dx = 0. \tag{3}$$

In the case of a beam with simply supported ends we shall seek the solution in the form

$$u = e_0\left(1-\frac{x}{L}\right)+\sum_{k=1}^{s} a_k \sin k\pi \frac{x}{L}, \qquad w=\sum_{k=1}^{s} f_k \sin k\pi \frac{x}{L}, \tag{4}$$

where $e(0)=e_0$ and the coefficients e_0, a_k, f_k are subject to variation. The integrals (2) will be calculated numerically; replacing these results into equations (3) we find the second derivatives $\ddot{e}_0, \ddot{a}_k, \ddot{f}_k$. The quantities e_0, a_k, f_k are evaluated according to the method of central finite differences (for getting a stable solution the time increment Δt must be sufficiently small).

In order to demonstrate the efficiency of the proposed algorithm several numerical examples are examined. With the purpose to establish chaotic effects the deflection history, phase and power spectrum diagrams are put together. It turned out that chaotic motion of the beam may exist, especially in the initial phase of motion; as to the long term motion then it transitates to periodic vibrations of smaller amplitude.

References

1. Lepik Ü., Dynamic response of elastic-plastic pin-ended beams by Galerkin's method. *Int. J. Solids Structures,* **31**, (1994), 3249-3260.
2. Lepik Ü., Elastic-plastic vibrations of a buckled beam. *Int. J. Non-Linear Mech.,* **30**, (1995), 129-139.
3. Symonds P.S. and Yu T.X., Counterintuitive behavior in a problem of elastic-plastic beam dynamics. *ASME J. Appl. Mech.,* **52**, (1985), 517-522.
4. Symonds P.S. and Qian Y., Aspects of chaotic response of an elastic-plastic beam to impulsive loading. Proceedings of the IUTAM and ISIMM Symposium on Anisotropy, Inhomogencity and Nonlinearity in Solid Mechanics (University of Nottingham, 1994), Kluwer, 1995.